Subject: Mathematics

General comments about the subject

Guideline Mathematics Linz International School (for examinations from 2014 onwards)

Mathematics is a compulsory subject for all students at the Linz International School. The number of lessons per week varies from grade to grade[1].

Students up to grade 6 are taught as whole class. There are additional Crash courses offered in the afternoons for those International students that enter school during the term to guarantee that they can catch up with the topics as soon as possible[2].

The Maths course at the Linz International School is based on the Austrian syllabus and includes all basic skills that Austrian students must cover as our school is state funded.

All these topics are part of the requirements of the IBO[3] for the final examinations at the end of grade 8 as well. There is only one topic for the SL course that is taught in addition (regression/ correlation) according to the new International syllabus (valid for IB examinations after 2014).

Detailed information on IB courses in grade 7 and 8:

In grade 5 students must choose their IB courses in Mathematics. We offer both, **standard (=SL) and higher level(=HL) Mathematics,** for the IB examinations in May and November. There is no course system as in many other IB schools where students can work on different chapters like algebra or geometry. At our school the Maths course lasts the whole school year and includes all different areas required to cover IB and Austrian syllabus.

[1] See LISA leaflet „Stundentafel“ or homepage

[2] The academic standard of Mathematics is quite high compared to international levels.

[3] =International Baccalaureate Organization

IB Students must have

1. **a GDC (=graphic display calculator) appropriate for the examinations in grade 8 (e.g. TI 84 plus or TI inspire[4])**
2. formula booklet from grade 7 onwards[5]
3. an IB Maths book from grade 6 onwards[6]
   (provided from schoool)

SL course: SL Students have three lessons of Mathematics in grade 7 and 8. They cover all topics they need for their IB papers at the end of year 8. There are two papers: one without a GDC and the second with GDC. Both papers include long and short respond questions and last for 1.5 hours.

Furthermore each student has to work on two portfolio (=PF)[7] tasks during the course individually. They can achieve 40 points the most which count to their final mark for their certificate in the end.

HL course: HL Students have a double lesson Maths in the second term of grade 6 in addition to the 3 lessons with the other students aiming for SL certificates. HL students have 4 lessons per week in grade 7 and 8.

They get their own teacher and are separated from the SL course. Certain core topics are equal to SL topics but taught with more depth (e.g. calculus: HL students must know implicit differentiation, related rates, differential equations, more integration methods than SL students etc.)

[4] Please ask one of the Maths teachers or look at the list on the IBO homepage before you buy a GDC to ensure you don't get a wrong model!
[5] Provided from school!
[6] Provided from school!
[7] Portfolio tasks are changed from the IBO on a regular base and handed out by the teachers at appropriate times during grades 7 and 8

A main advantage of this course is that the students have a brilliant work atmosphere as they are a group of between 5 and 9 students on average. So they can work according their interest and ability level with teacher input as required because of the small group size. Another advantage is that HL students have an option "statistics".

This is a topic that is the base of many courses at university (Chi squared test, hypothesis tests etc.).

HL students have **3 papers** for the examinations at the end of their course in year 8: paper one and two last for 2 hours each. And have similar structure and content than SL papers.

Paper 3 last for 1 hour and is based on the option "statistics" only. They can use formula booklet and GDC for this paper.

HL must work on two PF (=portfolio) that are different to SL tasks.

IB papers are sent to examiners of the IBO, they are marked externally. Whereas all other work of students during the courses (e.g. tests, mock exams, portfolio tasks[8],…) are marked internally by the teacher.

Generally it is wise to ask the Maths teacher which course is suitable for your child. They need insight and a love for the beauty of Maths to really be able to enjoy a HL Maths course. A mark better than a "Befriedigend" in the report card of 5th grade is also a school regulation to decrease the chances for an inappropriate choice of courses.

The following syllabus is for the **Mathematics Standard level course** at the Linz International School. The Higher level syllabus is attached at the end.

[8] Samples are sent to an official IBO moderator though to guarantee fair correction!

Year 1

(4 Maths lessons per week/ age[9] 10-11):

1.1 **Number and Algebra:**

- set of **natural numbers** (addition, subtraction, multiplication, division – last two for larger numbers as well)
- **Unit conversions** (length, time, weight, areas and volumes) to allow solving of more complex word problems
- set of **positive rational numbers** (decimal numbers: addition, subtraction, multiplication, division- also including leading figure estimation, checks and mental arithmetic!); using number rays for presentation; fractions; word problems
- rules for calculations with all four operation (=BODMAS); including word problems;

1.2 **variables:**

- manipulation and word problem solving using **variables**, parameters and unknown;
- **formulas and equations** incl. word problem solving
- solving simple linear equations
- use and interpretation of formulas

1.3 **figures and solids:**

- use of **shapes** in reality to identify geometric mathematical shapes incl. identification of properties (application of basics from primary education to solve more complex figures and solids (=combined areas and volumes))
- **sketch** and label rectangles, squares, cuboids,… using appropriate tools (e.g. protractors,
- **scale drawing** (produce maps and plans of real life samples)
- **perimeter and area of rectangles, squares and combined shapes**
- **volume and surface** area of cuboids (including unit conversions!)
- naming, drawing and identifying **angles**
- unit „degrees" (meaning and use of protractor in this context!)
- simple **symmetrical figures** (identification)

1.4 **statistics**

- identification of simple **direct proportions** (e.g. distance- time)

[9] of students attending course as a rough estimate

Year 2

(4 Maths lessons per week/ age 11-12):

2.1 **number and unit conversion**:

- explore and revise set of **rational numbers (positive** number range only) includes work on more complex real life word problems
- calculations with **fractions**
- convert fractions to decimals and inverse operation
- rules of divisibility
- **percentages** (e.g. convert to decimals and fractions; word problems)
- **unit conversion** to be able to work on geometrical problems appropriate for age group

2.2 **variables**:

- use of variables to describe real life problems
- develop **equations and formulas** (includes word problems)
- solve simple **linear equations** in one unknown
- make one of the variables the subject of a formula
- interpretation of given formulas

2.3 shapes and solids

- know and understand properties of triangles, quadrilaterals and **regular polygons**
- sketch and construct these figures
- recognition of multiple solutions or impossibilities when constructing shapes
- construct congruent shapes and explanation
- **perpendicular and angle bisector constructions**
- calculation of area of shapes that are or can be split into rectangles
- calculations of **volumes of prisms** using applied problems

2.4 Statistics and modelling

- **direct and indirect proportions** (for solving examples and text questions)
- simple tasks to be develop by students, solve them and interpret them critically
- **relative frequency**
- graphical presentation of relative frequencies and critical interpretation of these graphs

Year 3

(3 Maths lessons per week/ age 12-13):

3.1 number and unit conversion

- working with rational numbers (e.g. presentation on the number ray, relations using <, > signs, use of coordinate grid)
- revision of rules for operations with rational numbers (addition, subtraction, multiplication, division)
- use of **BODMAS** rules to combine all 4 types of calculations (work on these examples must be done with and without **calculator**!)
- using **power notation** in calculations (e.g. x^3+5x^3), also for word problems
- use of **scientific notation** (e.g. 4×10^4)

3.2 variables

- **transformations of formulas** and terms (including an insight for the rules of calculation applied)
- create formulas in context with word problems (also including geometry)
- solving word problems with real life context (also including transformations of formulas to make a certain variable the subject)
- use of **graphical representation**
- solving of **linear equations** in one unknown

3.3 shapes and solids

- increase and decrease of shapes using ratios and similarities
- recognition of **similar figures** and interpretation
- use and understanding of formulas to calculate the **area of triangles and quadrilaterals**
- inverse problems on areas (also text examples)
- draw **prisms and pyramids**
- calculation of volume, surface area and mass of real life objects which are of the shape of prisms and pyramids
- know and apply the **Theorem of Pythagoras**

3.4 statistics

- work on **linear growth and decay** processes (e.g. simple interest) using electronic supplies
- ev. recognition, application, understanding and graphical presentation of simple **functions**
- ev. knowledge, application and understanding of different **ways of presentations** for sets of data

Year 4

(3 Maths lessons per week/ age 13-14):

4.1 number and unit conversion

- recognition that the use of rational numbers is limited
- investigation on **irrational numbers** (includes use of electronic supply), includes critical reflection and discussion on useful rounding of numbers

4.2 variables

- revision of work on **variables, terms and equations**
- work with simple **algebraic fractions**
- solve linear equations in two unknown (also graphical interpretation of the set of solutions)
- further investigation work on **functions**

4.3 shapes and solids

- use and understanding of theorem of Pythagoras (includes applications for calculations in three dimensions)
- introduction and knowledge of formulas to calculate **circumference and area of a circle**
- knowledge of formula to calculate the area of an arc of a circle and the area of arc sections
- deduction of formulas for the calculation of **surface area and volume of cones, cylinders and spheres**

4.4 statistics

- investigation and presentation of **growth and decay** processes (includes use of electronic supply)
- investigation on and presentation of functions
- knowledge and application of basic statistical terminology (e.g. **mean, mode, median, quartile, relative frequency, box and whisker** (and other) diagrams)

Year 5

(3 Maths lessons per week/ age 14-15):

5.1 number and algebra

- Revision of extension of sets of numbers: natural numbers, integers, rational and irrational numbers
- **Powers** of ten to present very small and very large numbers in real life context (e.g. using applications in physics)
- Critical reflection on usefulness of results (rounding, precision of estimated values (e.g. for modelling: Does it make sense to round the number of trees in a forest accurately to the nearest unit?))
- **interpretation of terms and formulas** (give reasons for how transformations work using rules)
- investigation on prime numbers and divisibility

5.2 systems of equations

- **Solving linear and quadratic equations** in one unknown
- Solving systems of **linear equations in two unknown** including a geometrical approach and interpretation
- Applications of the techniques to solve real life word problems

5.3 functions

- **definition** of the term "**function**" and investigation on properties of a function in one variable (incl. ways of presentation: terms, tables, graphs)
- use of functions to **model** real life situations
- definition and investigation of **linear and nonlinear functions** (e.g. rational, hybrid, polynomial functions)
- using functions to model direct and indirect proportions
- word problems of real life situations that require functions to be solved
- transformations of curves (shrinking, enlarging, motions along x- and y- axis)

5.4 trigonometry

- **definition of sine,** cosine and tangent functions within one full turn on the unit circle
- use of trigonometry in **right angled triangles** (SOHCAHTOA)(also includes word problems)
- use of sine and cosine rule in **non right angled triangles** (also includes word problems)
- introduction of **polar coordinates**
- **transformations** of circular trigonometric functions (dilations etc.)
- trigonometric equations
- arc length, area of arc sector

5.5 vector calculation

- **addition, subtraction and multiplication of vectors in two dimensions**
- **geometrical** interpretation of the above operations in the coordinate grid
- **unit and normal vectors** in two dimensions
- **scalar product** and hence calculation of the angle between two vectors
- **equation of lines in vector form, parametric and Cartesian form**, relation of two lines in two dimensions (intersection, perpendicular lines, parallel lines)

Year 6

(3 Maths lessons per week/ age 15-16):

6.1 powers, roots and logarithms:

- definition of powers of natural , whole, rational and real numbers
- definitions of **roots and logarithms**
- deductions and proofs of **laws of calculation for powers, roots, logarithms** (incl. transformations of the terms using **algebraic skills**)

6.2 sequences and series:

- **general definition of sequences; recursions and explicit representation**
- properties (e.g. convergent series, limit, monotony,…)
- definition of the "Euler Number"
- **Arithmetic and Geometric** sequences and series. Recognition of parallels between arithmetic sequences and linear functions, as well as between geometric sequences and exponential functions
- Applications (e.g. **finances:** superannuation and compound interest)
- Binomial Theorem (incl. Pascal´s triangle)

6.3 equations, inequalities, systems of equations:

- simple **inequalities** (also use of GDC)
- Solving **3 equations in 3 unknown**

6.4. real functions:

- definitions, presentations and knowledge of properties of **power functions, exponential- and logarithm functions; circular trigonometric functions** (radian measure)
- **properties** of the above functions (domain, range, monotony, local and global stationary points, periodicity symmetry)
- **inverse functions**
- **rate of change** (average rate of change)
- Use of functions to describe continuous processes, **modelling** and critical interpretation of results and outcomes; discussions on limitations and assumptions of models (e.g. growth and decay functions)
- **Composite functions**

6.5 geometry

- revision of important definitions in 2 dimensions and transformation to **3 dimensions** (discussion on normal vectors and the limitations of the transformation)
- definition of the **cross product** (=vector product) and use to find normal/ perpendicular vectors in 3 dimensions; also: area of a triangle
- different types of equations of **lines and planes** in 3 dimensions
- **relationship**s between lines, planes, lines and planes (parallel, coincident, skew, intersecting)
- **angle** between line and planes,…
- solving word problems (more complex geometrical applications also including trigonometry)

6.6 probability

- revision of **basic statistics** (mean, variance, diagrams (box and whisker, cumulative frequency polygon,…))
- definition of randomly carried out experiments, events etc.
- probability and **relative frequency** ("Satz der großen Zahlen")
- rules of probability (**addition and multiplication rule, conditional probability**) and ways of presentation: **tree diagram, lattice diagram, Venn diagram,**…
- **Theorem of Bayes**, arrangements (incl. combinations, permutations etc.)

Year 7

(3 Maths lessons per week/ age 16-17)

7.1 calculus (differentiation)

- Definition and deduction of the "**instantaneous rate of change**" incl. gradient of secant and tangent (inc. tangents and normals to a curve at particular points)
- Differentiation from **first principle**
- **Rules** of differentiation (power-, product-, quotient- and chain rule)
- **Standard derivatives**
- **Sketching graph** of derivatives of a function
- Interpretation of second derivative as curvature of curve
- Applications: 1. **kinematics**
  2. **optimization**
  3. **curve sketching (polynomial, rational and general functions**)

7.2. probability distributions

- Definition of the term "**random variable**"
- **definition, presentation, E(X) and V(X) of discrete distributions**
- **multiples of random variables** (e.g. E(3X-4))
- **Binomial distribution**
- Word problems

Year 8

(3 Maths lessons per week/ age 17-18):

8.1. calculus (Integration)

- definition of **antiderivatives**
- definition of **integral** (as approximation of sum of area of infinitely many rectangular areas)
- definite and indefinite integrals
- recognition of connection between differentiation and integration
- **interpretation of integration** (area under curve, area between two curves, solids of revolution)

8.2 probability distributions

- definition of **continuous** random variables and distributions
- **Normal distribution**

8.3 revision of ALL topics

Syllabus HL course Mathematics:

HL students will cover the **same core topics than SL students with the following additions:**

Grade 6: (double lesson of Maths in summer term)

- **inverse circular trigonometric functions** (incl. transformations)
- **remainder-, factor theorem, polynomial long division**
- **matrices**
- start with differentiation (which SL do in 7th grade)

Grade 7: (4 Maths lessons per week)

- differentiation:
  1. implicite differentiation
  2. related rate of change
  3. derivatives of inverse trigonometric functions

- **complex numbers:**
  - basics
  - Argand diagram
  - polar vs. Cartesian form
  - Polynomials over the complex field
  - Calculations with polar coordinates

  (incl. Fundamental Theorem of Algebra De Moivre´s Theorem and roots, conjugate root theorem)

- **proof by induction**
- discrete distributions:
  1. **Poisson distribution**
  2. **Hypergeometric distribution**
  3. **Geometric distribution**
  4. **Negative Binomial Distribution**
  5. **uniform distribution**

Grade 8: (4 Maths lessons per week)

- **integration:** continuous probability density functions (mean, mode, median, variance, standard deviation)
- continuous probability distributions:
  1. uniform distribution
  2. Exponential Distribution

- **option: "statistics"**
  - sampling
  - **hypothesis tests** (for all distributions!) also including Type I and Type II errors
    Also including z- and t- tests
  - Normal distribution as approximation of Binomial distribution
  - **confidence intervals**
  - **chi² test** (both goodness of fit and independence test
  - multiples of random variables
  - interpretation of results